

The Abel Prize 2016

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The Norwegian Academy of Science and Letters has decided to award the Abel Prize for 2016 to **Sir Andrew J. Wiles**, University of Oxford

for his stunning proof of Fermat's Last Theorem by way of the modularity conjecture for semistable elliptic curves, opening a new era in number theory.



Sir Andrew J. Wiles

Fermat's Last Theorem: The equation

$$x^n + y^n = z^n$$

has no solution in positive integers for $n > 2$.



Pierre de Fermat



Leonhard Euler



Sophie Germain

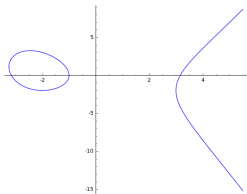


Ernst Kummer

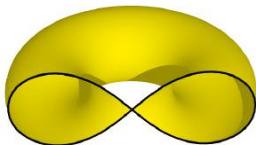
Elliptic Curve: Solutions to a cubic equation

$$y^2 + a_1xy + a_3y = x^3 + a_2x^2 + a_4x + a_6$$

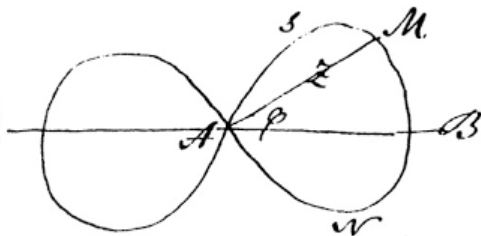
in two variables x and y .



Real solutions



Complex solutions

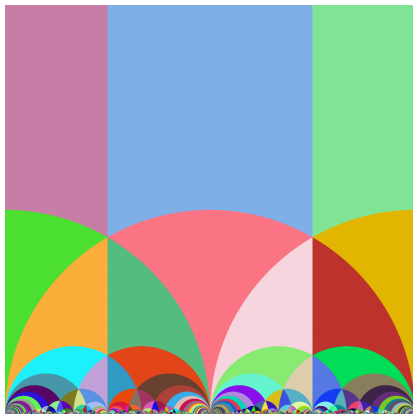


Abel's drawing

Modular form: Highly symmetric analytic function

$$f: \mathcal{H} \longrightarrow \mathbb{C}$$

defined on the upper half of the complex plane.



Fundamental region for modular curve $X(1) = \mathcal{H}/SL_2(\mathbb{Z})$

Modularity Conjecture: Every elliptic curve defined over the rational numbers can be parametrized by a modular curve

$$X_0(N) \longrightarrow E.$$



Yutaka Taniyama



Goro Shimura



André Weil

If $a^\ell + b^\ell = c^\ell$ (with $\ell \geq 5$ a prime) then the elliptic curve

$$y^2 = x(x - a^\ell)(x + b^\ell)$$

is not modular. Hence Modularity Conjecture $\implies$ Fermat's Last Theorem.



Gerhard Frey



Jean-Pierre Serre



Kenneth Ribet

I think I'll stop here.



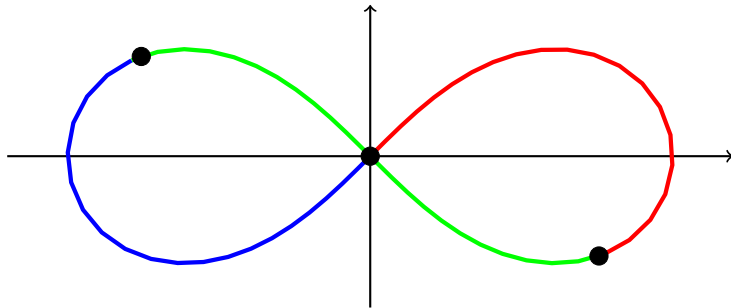
Andrew Wiles, June 23rd 1993

Galois representations:

$$\begin{array}{ccc} & & GL_2(\mathcal{O}) \\ & \nearrow \rho & \downarrow \\ \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) & \xrightarrow{\rho_0} & GL_2(\bar{\mathbb{F}}_p) \end{array}$$



Barry Mazur

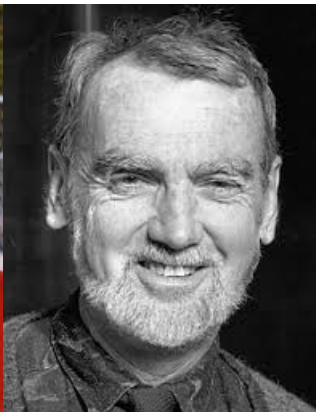


Points of order $p = 3$ on the lemniscate

Complete intersection / Modularity in $GL_2(\mathbb{F}_3)$:



Richard Taylor

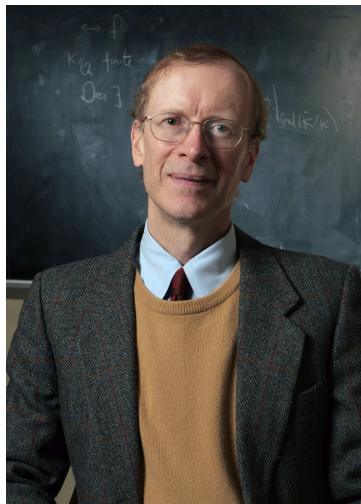


Robert Langlands



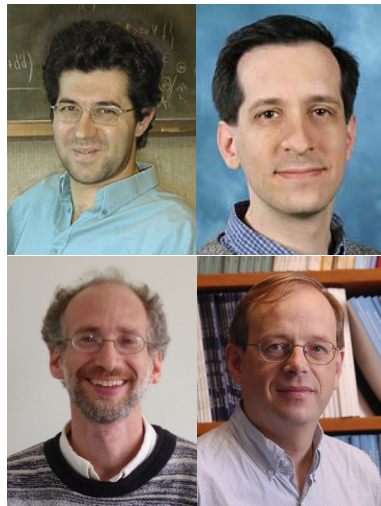
Jerrold Tunnell

The 3-5 switch:



Wiles

The full modularity conjecture:



Breuil, Conrad, Diamond, Taylor

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Page 4: Fermat: Wikimedia commons; Euler: Wikiquote; Germain and Kummer: MacTutor History of Mathematics Archive.

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Page 6: Fundamental region: Tom Womack.

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