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Research statement

I work on numerical methods for partial differential equations, especially finite element methods. I'm interested in structure preserving discretizations. I both develop new algorithms and study their properties theoretically.

Main achievements. My PhD was on integral equations for wave equations, especially Maxwell's equations for electromagnetics. I renewed the numerical analysis of boundary element methods, through the use of discrete Hodge decompositions. I introduced preconditioners based on Calderon formulas. This project was brought to completion during my post doc, by the introduction of dual finite element complexes [11].

After that I moved my attention to non-linear geometric wave equations from theoretical physics:

– For the Einstein equations of General Relativity I have linked Regge Calculus to finite element methods, inserting the spaces in a complex and proving convergence for eigenvalue problems [17].

– For the Yang-Mills equations, and other gauge theories such as the Maxwell-Klein-Gordon equations, I have studied constraint preserving schemes. In particular the Lattice Gauge Theory formalism developed for Quantum Chromodynamics was combined with finite element methods to derive new gauge invariant schemes on simplices, for which Noether type theorems apply and give conservation laws [26].

Some of my work contributes to the foundations of "finite element exterior calculus", through in particular the design of uniformly bounded projections that commute with the differential operators at hand [14]. I have also introduced a new perspective on finite elements, called "finite element systems", that provides a unified picture of old and new methods, expressed in the language of categories. In particular cohomological properties of discrete complexes are clarified (de Rham theorems). It has been used to design new finite element complexes for smooth differential forms, including the Stokes equation [33], and for Riemannian geometry, including elasticity problems [37].

Moreover. The framework of finite element systems has been developed in several other directions: to accomodate polyhedral meshes [13], contain upwinded basis functions [28], have minimal dimension [30] or account for known elements [29].

In order to study the various algorithms I have extended several functional-analytic tools to a Galerkin setting. This includes Fredholm theory [4], stability and compactness estimates on Hodge decompositions [12], div-curl lemmas in the sense of Murat-Tartar [9] [23], Sobolev injections [21] and fractional order Sobolev spaces [31].

A major thread has been to import concepts from differential geometry to a finite element setting. I was early to leverage the language of differential forms as well as metrics, connections and curvatures on vector bundles, in numerical problems from engineering and physics.

Along the way I have made some other contributions:

– Integral equations: [2] [3] [7] [5] [6] [16].

– Regge calculus: [8] and preprint [1].

– Gauge theories: [10] [15] [18] [19] [22] and preprint [3].

– Finite elements: [20] [27] [25] [32] [35] [36] and preprints [2] [4].

– Other topics: [1] [34] .

Citations. See Google Scholar or MathSciNet.

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